Treatment Effects: What works for Whom?

Spyros Konstantopoulos
spyros@msu.edu
Michigan State University

Prepared for the IES Summer Research Training Institute June 2022

Main Treatment Effects

- In Experiments Typically We Calculate an Average Treatment Effect

- We Are Interested in Estimating Treatment Effects for *All* Individuals in the Sample

- We Compute the Average Difference Between Treatment and Control Groups in Outcomes of Interest (e.g., Achievement) for *All* Individuals (Main Effect)

Main Treatment Effects

- In the Simplest Case We Can Conduct a t-test for Independent Samples to Examine the Significance of the Main Treatment Effect

- Alternatively (and Equivalently) We Can Run a Simple Regression or ANOVA (with a Dummy Variable for the Treatment). The t-test is the Same (Assuming the Variances in the Two Groups Are Equal)

- We Can Run ANCOVA or Multiple Regression to Include Covariates

- In Nested Designs we Use Multilevel Models (HLM)

Does the Treatment Have the Same Effect On All Groups of Individuals?

- One Fundamental Objective of U.S. Education is to Close the Achievement Gap Among Important Student Groups
- School Interventions Have Frequently Dual Objectives:
  - Increase Student Outcomes (e.g., Achievement) for All Students
  - Reduce the Achievement Gap
- Note that Decreasing the Achievement Gap Suggests that the Treatment Effect May Not Be the Same for All Students
- Some Students May Benefit More from Treatments than Others

Does the Treatment Have the Same Effect On ALL Groups of Individuals?

- Interventions May Help Reduce the Gender, Race/Ethnicity, SES, ELL, and Low- High-Achievement gap

- Interventions May Help Reduce Differences between Schools (e.g., Urban and Rural schools)

- This Could Be a Byproduct of the Intervention or Could be one of the Main Objectives of the Study

- For example, the Objective Is to Reduce/Close the Achievement Gap at the Individual or School Level

Differential Effects

- Exposure to Treatment Differs for Various Groups of Individuals

- The Effectiveness of the Treatment Varies Across these Groups

- These Are Called Differential Effects of the Treatment for Certain Groups of Individuals

Differential Effects

- Examples:
- Low Achievers May Benefit More from Small Classes
- Low Achievers May Benefit More from Effective Teachers
- Low Achievers May Benefit More from Data Driven Assessments
- Assessment Programs May Be More Beneficial To Rural Schools
- These Differential Effects Are Introduced in Regression Models as Statistical Interactions
- Alternatively, to Determine Treatment Effects at Different Levels of Continuous Outcomes (e.g., Achievement) One Can Use Quantile Regression

Interaction Effects

- Interaction Effects Are a Related Notion to Differential Effects
- The Idea Is that the Treatment Interacts with Individual Characteristics (e.g., Low SES).
- Through the Interaction the Treatment Could Be Maximized for a Specific Group of Individuals (e.g., low SES students) or Clusters (e.g., Rural Schools) on a Specific Outcome (e.g., Achievement)

Interaction Effects

- Pioneering Work by Cronbach and Snow (1977) Discussed Aptitude-Treatment Interactions in Education
- The Idea is that a Treatment (e.g., Highly Structured Instruction) May Benefit Some Students (e.g., Low Achievers) More than Others

Moderator Effects

- Same Notion as Interaction or Differential Effects
- Variables that Interact with Treatments Are Called Moderators and Indicate the Degree to Which the Treatment Effect on an Outcome Depends on the Moderator (Baron & Kenny, 1986)
- Moderator Variables Can be Categorical (e.g., Gender, Race/Ethnicity), Ordinal (e.g., SES, Likert Scale), or Continuous (e.g., Ability-Prior Achievement, Teacher Experience)
- Analytically, Interaction or Moderator Effects Are Introduced in Linear Regression Models as Statistical Interactions

Moderator Effects

- The Moderator May Affect the Direction and the Magnitude of the Treatment Effect
- That Is, the Moderator Variable Will Change the Strength of the Association Between Treatment and Outcome
- The Moderator Can Amplify or Reverse the Treatment Effect
- The Question of Interest Is How Universal Is the Effect?
- The Moderator Is Selected In Accord with the Researcher's Interests (e.g., Research On Gender, Race, SES, Achievement, School Differences, Wages, etc.)

What is a Statistical Interaction?

- Suppose We Are Interested in Examining Whether Small Classes Increase Achievement for Low SES (e.g., Students Eligible for Free or Reduced-Price Lunch) More than Other Students

- We Can Construct Two Binary Variables for Small Class and for Low SES Status and Create a Statistical Interaction by Multiplying the Two Variables

- In this Example Low SES Status Is a Moderator Variable. The Idea is that the Effect of Small Class is Different for Low SES than for Higher SES Students

Modeling Statistical Interaction

- The Simplest Form of Interactions is Two-Way Interactions Between Two Variables (That's What We Discuss Here). A Pair of Variables Creates One Interaction Term

- Three-Way Interactions Are Between Three Variables (More Complicated Model). Three Variables Create Three Two-Way Interaction Terms and One Three-Way Interaction

Modeling Statistical Interaction

- Interactions Can Be Constructed between
  - Continuous with Binary Variable (e.g., School Composition and Private-Public School)
  - Continuous with Continuous Variable (e.g., Teacher Effectiveness and Professional Development)
  - Dummy with Dummy Variable (e.g., Small Class and Low-High SES)

- Next, We Will Discuss Interactions with a "Treatment" (Binary Variable)

- To Model Interactions, We Include in the Regression Equation *All* Main Effects (e.g., Small Class and Low SES) and the Two-Way Interaction (the Product of the Two Variables)

Modeling Statistical Interaction in Regression

- Suppose Treatment (T) is Dummy Variable (e.g., Small Class = 1, Else = 0) and Moderator (M) is Dummy Variable (e.g., Low SES Student = 1, Else = 0)
- The Simplest Way to Model the Interaction Effect Is

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 M_i + \beta_3 T_i M_i + \varepsilon_i$$

- Where y Is Outcome, ε Is Residual, and β's Need to Be Estimated (β_3 Is the Coefficient of Interest)

Hypothesis Testing

- The Null Hypothesis States that the Interaction Term is Zero
- The Alternative Hypothesis States that the Interaction Term is Different than Zero
- In this Case the Most Important Coefficient Is the Interaction Effect
- When the t-test Is Significant the Treatment Effect Is Changed by the Moderator

Class Size Example: Data

- Project STAR is a Longitudinal Field Experiment

- Students and Teachers Were Randomly Assigned to Small and Regular Size Classes Within Grades /Schools

OLS Analysis

Regression, Population Model:

$$y_i = \beta_0 + \beta_1 SMCLASS_i + \beta_2 LowSES_i + \beta_3 INT_i + \varepsilon_i$$

Where *INT* Represents the Interaction Term.
Suppose *Y* Represents Mathematics Scores

Results

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	.260	.022		11.948	.000	.217	.303
	SMALL CLASS	.141	.038	.066	3.763	.000	.068	.215
	LOW SES	-.630	.031	-.317	-20.365	.000	-.691	-.570
	LOW SES SMALL CLASS INTERACTION	.038	.055	.014	.695	.487	-.069	.145

a. Dependent Variable: MATHEMATICS

Interpreting Statistical Interactions

- Intercept/Constant: Average Math Achievement of High SES Students in Regular Classes
- Small Class Coefficient: Average Math Difference Between Small and Regular Classes for High SES Students
- Low SES Coefficient: Average SES (Low versus High) Math Achievement Gap in Regular Classes
- Interaction Coefficient: The Low-High SES Gap in Mathematics Is Reduced in Small Classes by an Average of 0.038 Points (Compared to the Regular Size Classes Gap), (Not Statistically Significant)
- OR the Treatment Effect (the Small-Regular Size Gap) for Low SES Students Is Larger (than that for High SES Students) by an Average of 0.038 Points in Mathematics (Not Statistically Significant)

Centering Main Variables

- Some Researchers Center the Treatment and Moderator Variables at their Means and then Compute the Product Term
- Centering Helps with Collinearity and Affects Only Estimates of Main Effects and their Standard Errors (Not Interactions)
- Standard Errors of the Main Effects Become Smaller Typically and the Intercept Is Different as Well
- Centering Seems more Natural when Continuous Variables Are Involved

Centering Main Variables

- The Previous Model Becomes:

$$y = \beta_0 + \beta_1(T - \overline{T}) + \beta_2(M - \overline{M}) + \beta_3(T - \overline{T})(M - \overline{M}) + \varepsilon$$

- When $\beta_3 = 0$ then

$$y = (\beta_0 - \beta_1\overline{T} - \beta_2\overline{M}) + \beta_1 T + \beta_2 M + \varepsilon$$

- which Indicates that the Slopes Are the Same as without Centering, but the Intercept Changes and Complicates the Interpretation

Centering Main Variables

- When $\beta_3 \neq 0$ the Model Becomes:

$$y = \left(\beta_0 - \beta_1\overline{T} - \beta_2\overline{M} + \beta_3\overline{TM}\right) + \left(\beta_1 - \beta_3\overline{M}\right)T + (\beta_2 - \beta_3\overline{T})M + \beta_3 TM + \varepsilon$$

- which Shows that the Interaction Term Does not Change but the Intercept and the Slopes Change and Complicate the Interpretation

Results: Centered Variables

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	.008	.013		.656	.512	-.017	.033
	SMALL CLASS CENTERED AT MEAN	.159	.027	.075	5.850	.000	.106	.213
	LOW SES CENTERED AT MEAN	-.618	.025	-.311	-24.247	.000	-.668	-.568
	LOW SES SMALL CLASS INTERACTION WITH CENTERED VARIABLES	.038	.055	.009	.695	.487	-.069	.145

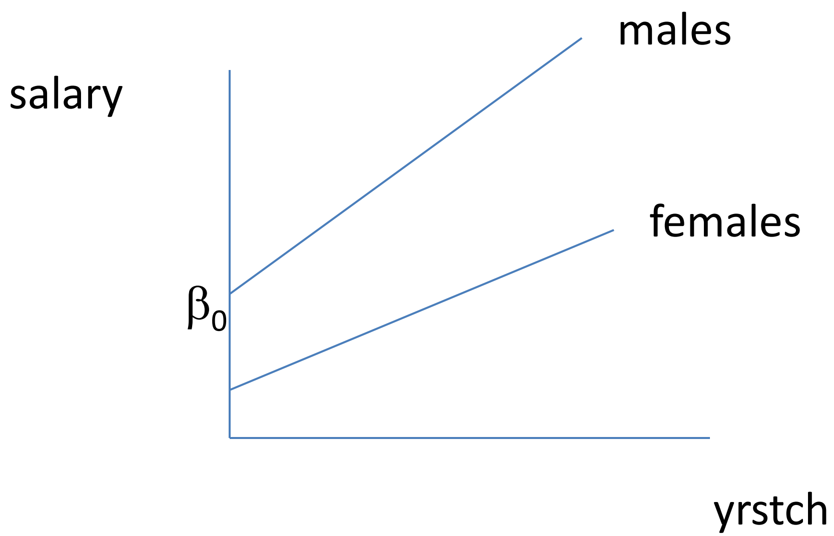
a. Dependent Variable: MATHEMATICS

- Notice that the SEs of the Main Effects are now Smaller

Another Example of an Interaction

- Does Being Male Faculty Have a Greater Impact on Salary with more Years of Experience?
- Stated differently:
  - Do Male Faculty Earn Higher Salaries than their Female Peers over Time ("Treatment" Here Is Gender)?
  - For Every Additional Year of Experience, Individuals Earn more, on Average (Experience Is the Moderator Variable)
  - Do Male Faculty Get Higher Increases for each Additional Year of Experience Compared to their Female Peers?

Allows the Male and Female Lines to Have Different Slopes



- Yrstch is Years of Experience

Add an Interaction Term to Capture the Added Effect

- This Model Assumes that the Effect of Years of Experience Is the Same for Men and Women (Experience is a Control Variable)

$$salary = \beta_0 + \beta_1 yrstch + \delta_1 female + \varepsilon$$

- The Model Below Relaxes this Assumption and Allows Experience to Interact with Gender

$$salary = \beta_0 + \beta_1 yrstch + \beta_2 female + \beta_3 (yrstch * female) + \varepsilon$$

Interaction: Null Hypothesis

- The Impact of Years of Experience on Faculty Salary is the Same for both Genders

Effect of Specific Variables Harder to Interpret with Interactions

- In the Model

$$salary = \beta_0 + \beta_1 yrstch + \beta_2 female + \beta_3 (yrstch * female) + \varepsilon$$

- the Effect of Experience for Males Is the Estimate of β_1 and at Average Levels of Experience, the Estimated Effect of Being Female Is

$$\hat{\beta}_2 + \hat{\beta}_3 * \overline{yrstch}$$

Another Example

- Do Gender Differences in Wages Depend on Years of Education?

- Gender Is the “Treatment” and Years of Education Is the Moderator Variable

What Do These Graphs Indicate?

Interactions in Multilevel Designs/Models

Modeling Statistical Interaction: Two-Level CRD - 1

- Outcome is Math Scores
- Treatment and Moderator at Level 2
- Suppose Treatment (T) is a Dummy Variable (e.g., School Intervention = 1, Else = 0) and Moderator (M) is Dummy Variable (e.g., Rural School = 1, Urban = 0)
- The Simplest Way to Model the Interaction Effect Is

$$L-1 \quad y_{ij} = \beta_{0j} + \varepsilon_{ij}$$

$$L-2 \quad \beta_{0j} = \gamma_{00} + \gamma_{01}T_j + \gamma_{02}M_j + \gamma_{03}T_jM_j + \eta_{0j}$$

Modeling Statistical Interaction: Two-Level CRD – 1 Example

- Treatment Represents Interim Assessments in Grades K-8 (School Level)
- Moderator Represents Rural and Urban Schools (School Level)

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	-.238060	.149575	33.732	-1.592	.121	-.542123	.066003
treatment	-.050011	.200831	33.835	-.249	.805	-.458223	.358200
rural	.260266	.173433	34.263	1.501	.143	-.092093	.612625
treatrural	.175248	.229866	34.226	.762	.451	-.291782	.642278

a. Dependent Variable: Standardized values of (math) .

Interpreting Statistical Interactions

- Intercept/Constant: Average Math Achievement In Control Schools In Urban Schools
- Treatment Coefficient: Average Math Difference Between Treatment and Control Schools in Urban Schools
- Rural School Coefficient: Average Rural versus Urban Schools Math Achievement Gap in Control Schools
- Interaction Coefficient: The Rural-Urban School Gap in Mathematics Is Larger in Treatment Schools by an Average of 0.175 Points (Compared to the Control Schools Gap) (Not Statistically Significant)
- OR the Treatment Effect in Rural Schools Is Larger (than that in Urban Schools) by an Average of 0.175 Points in Mathematics (Not Statistically Significant)

Modeling Statistical Interaction: Two-Level CRD - 2

- Treatment at Level 2 and Moderator at Level 1 (Fixed at Level 2)
- This Is a Cross-Level Interaction
- Suppose Treatment (T) is a Dummy Variable (e.g., School Intervention = 1, Control = 0) and Moderator (M) is a Dummy Variable (e.g., Female = 1, Male = 0)
- The Simplest Way to Model the Interaction Effect Is

$$L-1 \qquad y_{ij} = \beta_{0j} + \beta_{1j} M_{ij} + \varepsilon_{ij}$$

$$L-2 \qquad \beta_{0j} = \gamma_{00} + \gamma_{01} T_j + \eta_{0j}$$

$$L-2 \qquad \beta_{1j} = \gamma_{10} + \gamma_{11} T_j$$

Modeling Statistical Interaction: Two-Level CRD - 2

- The Mixed Effects Model is:

$$y_{ij} = \gamma_{00} + \gamma_{01} T_j + \gamma_{10} M_{ij} + \gamma_{11} T_j M_{ij} + \eta_{0j} + \varepsilon_{ij}$$

- Where γ_{11} Represents the Cross-Level Two-Way Interaction

Modeling Statistical Interaction: Two-Level CRD – 2 Example

- Treatment Represents Interim Assessments in Grades K-8 (School Level)
- Moderator Represents Female and Male Students (Student Level)

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	-.022597	.065169	56.674	-.347	.730	-.153112	.107918
treatment	.070702	.083267	56.991	.849	.399	-.096037	.237441
female	.003545	.019893	20869.970	.178	.859	-.035447	.042537
treatfemale	.004697	.026568	20868.169	.177	.860	-.047378	.056773

a. Dependent Variable: Standardized values of (math) .

Interpreting Statistical Interactions

- Intercept/Constant: Average Math Achievement In Control Schools for Male Students
- Treatment Coefficient: Average Math Difference Between Treatment and Control Schools for Male Students
- Female Coefficient: Average Female-Male Math Achievement Gap in Control Schools
- Interaction Coefficient: The Female-Male Gap in Mathematics Is Larger in Treatment Schools by an Average of 0.005 Points (Compared to the Control Schools Gap) (Not Statistically Significant)
- OR the Treatment Effect for Female Students Is Larger (than that for male Students) by an Average of 0.005 Points in Mathematics (Not Statistically Significant)

Modeling Statistical Interaction: Three-Level CRD - 1

- Students Nested Within Classes Within Schools
- Treatment and Moderator at Level 3
- Suppose Treatment (T) is Dummy Variable (e.g., School Intervention = 1, Else = 0) and Moderator (M) is Dummy Variable (e.g., Private School = 1, Else = 0)
- The Simplest Way to Model the Interaction Effect Is

$$L-1 \quad y_{ijk} = \beta_{0jk} + \varepsilon_{ijk}$$

$$L-2 \quad \beta_{0jk} = \gamma_{00k} + \eta_{0jk}$$

$$L-3 \quad \gamma_{00k} = \delta_{000} + \delta_{001} T_k + \delta_{002} M_k + \delta_{003} T_k M_k + \xi_{00k}$$

Modeling Statistical Interaction: Three-Level CRD - 2

- Treatment at Level 3 and Moderator at Level 2 (Fixed at Level 3)
- This Is a Cross-Level Interaction
- Suppose Treatment (T) is Dummy Variable (e.g., School Intervention = 1, Else = 0) and Moderator (M) is Dummy Variable (e.g., Teacher Certified = 1, Else = 0)
- The Simplest Way to Model the Interaction Effect Is

$$L-1 \quad y_{ijk} = \beta_{0jk} + \varepsilon_{ijk}$$

$$L-2 \quad \beta_{0jk} = \gamma_{00k} + \gamma_{01k} M_{jk} + \eta_{0jk}$$

$$L-3 \quad \gamma_{00k} = \delta_{000} + \delta_{001} T_k + \xi_{00k}$$

$$L-3 \quad \gamma_{01k} = \delta_{010} + \delta_{011} T_k$$

Modeling Statistical Interaction: Three-Level CRD - 2

- The Mixed Effects Model is:

$$y_{ijk} = \delta_{000} + \delta_{001} T_k + \delta_{010} M_{jk} + \delta_{011} T_k M_{jk} + \xi_{00k} + \eta_{0jk} + \varepsilon_{ijk}$$

- Where δ_{011} Represents the Cross-Level Two-Way Interaction

Modeling Statistical Interaction: Three-Level CRD - 3

- Treatment at Level 3 and Moderator at Level 1 (Fixed at Levels 2 and 3)
- This Is Also a Cross-Level Interaction
- Suppose Treatment (T) is Dummy Variable (e.g., School Intervention = 1, Else = 0) and Moderator (M) is Dummy Variable (e.g., Low SES = 1, Else = 0)
- The Simplest Way to Model the Interaction Effect Is

$$L-1 \qquad y_{ijk} = \beta_{0jk} + \beta_{1jk} M_{ijk} + \varepsilon_{ijk}$$

$$L-2 \qquad \beta_{0jk} = \gamma_{00k} + \eta_{0jk}$$

$$L-2 \qquad \beta_{1jk} = \gamma_{10k}$$

$$L-3 \qquad \gamma_{00k} = \delta_{000} + \delta_{001} T_k + \xi_{00k}$$

$$L-3 \qquad \gamma_{10k} = \delta_{100} + \delta_{101} T_k$$

Modeling Statistical Interaction: Three-Level CRD - 3

- The Mixed Effects Model is:

$$y_{ijk} = \delta_{000} + \delta_{001}T_k + \delta_{100}M_{ijk} + \delta_{101}T_kM_{ijk} + \xi_{00k} + \eta_{0jk} + \varepsilon_{ijk}$$

- Where δ_{101} Represents the Cross-Level Two-Way Interaction

- The Model Can Become More Complicated and Include Moderators At All Levels

Modeling Statistical Interaction: Two-Level BRD

- In BRD Interactions Can Be Modeled as Fixed (e.g., Two-Way Interactions) or Random (e.g., Treatment by Class or School Interaction). Students Nested Within Schools
- Treatment and Moderator at Level 1 (Both Fixed at Level 2)
- Suppose Treatment (T) is Dummy Variable (e.g., Small Class = 1, Else = 0) and Moderator (M) is Dummy Variable (e.g., Female = 1, Else = 0)

$$L-1 \quad y_{ij} = \beta_{0j} + \beta_{1j}T_{ij} + \beta_{2j}M_{ij} + \beta_{3j}T_{ij}M_{ij} + \varepsilon_{ij}$$

$$L-2 \quad \beta_{0j} = \gamma_{00} + \eta_{0j}$$

$$L-2 \quad \beta_{1j} = \gamma_{10}$$

$$L-2 \quad \beta_{2j} = \gamma_{20}$$

$$L-2 \quad \beta_{3j} = \gamma_{30}$$

Example

Two-Level Model:

$$Y_{ij} = \beta_{0j} + \beta_{1j}\text{SMCLASS}_{ij} + \beta_{2j}\text{LowSES}_{ij} + \beta_{3j}\text{Interaction}_{ij} + \varepsilon_{ij}$$

$$\beta_{0j} = \gamma_{00} + \eta_{0j}$$

$$\beta_{1j} = \gamma_{10}$$

$$\beta_{2j} = \gamma_{20}$$

$$\beta_{3j} = \gamma_{30}$$

Results

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	.202218	.048519	90.220	4.168	.000	.105829	.298606
small3	.142518	.035817	5414.423	3.979	.000	.072302	.212734
lowses3	-.490843	.032475	5419.302	-15.114	.000	-.554508	-.427179
sesmal	.052090	.051703	5404.219	1.007	.314	-.049269	.153450

a. Dependent Variable: MATHEMATICS.

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		.774205	.014925	51.873	.000	.745498	.804017
Intercept [subject = schid3]	Variance	.128409	.024088	5.331	.000	.088903	.185470

a. Dependent Variable: MATHEMATICS.

Modeling Statistical Interaction: Two-Level BRD - 1

- In BRD Interactions Can Be Modeled as Fixed (e.g., Two-Way Interactions) or Random (e.g., Treatment by Class or School Interaction). Students Nested Within Schools
- Treatment and Moderator at Level 1: Treatment is a Random Effect at Level 2
- Suppose Treatment (T) is Dummy Variable (e.g., Small Class = 1, Else = 0) and Moderator (M) is Dummy Variable (e.g., Female = 1, Else = 0)

$$L-1 \quad y_{ij} = \beta_{0j} + \beta_{1j} T_{ij} + \beta_{2j} M_{ij} + \beta_{3j} T_{ij} M_{ij} + \varepsilon_{ij}$$

$$L-2 \quad \beta_{0j} = \gamma_{00} + \eta_{0j}$$

$$L-2 \quad \beta_{1j} = \gamma_{10} + \eta_{1j}$$

$$L-2 \quad \beta_{2j} = \gamma_{20}$$

$$L-2 \quad \beta_{3j} = \gamma_{30}$$

Modeling Statistical Interaction: Two-Level BRD - 1

- The Mixed Effects Model is:

$$y_{ij} = \gamma_{00} + \gamma_{10} T_{ij} + \gamma_{20} M_{ij} + \gamma_{30} T_{ij} M_{ij} +$$

$$\eta_{0j} + T_{ij} \eta_{1j} + \varepsilon_{ij}$$

- Where γ_{30} Represents the Two-Way Interaction and $T_{ij}\eta_{1j}$ Represents a Treatment by Level-2 (School) Interaction (Random Effect)

Example

Two-Level Model:

$$Y_{ij} = \beta_{0j} + \beta_{1j}\text{SMCLASS}_{ij} + \beta_{2j}\text{LowSES}_{ij} + \beta_{3j}\text{Interaction}_{ij} + \varepsilon_{ij}$$

$$\beta_{0j} = \gamma_{00} + \eta_{0j}$$

$$\beta_{1j} = \gamma_{10} + \eta_{1j}$$

$$\beta_{2j} = \gamma_{20}$$

$$\beta_{3j} = \gamma_{30}$$

Results

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	.195690	.049486	85.040	3.954	.000	.097299	.294081
small3	.156287	.053562	101.434	2.918	.004	.050040	.262535
lowses3	-.480828	.032889	5222.762	-14.620	.000	-.545303	-.416352
sesmal	.038680	.056528	3115.367	.684	.494	-.072155	.149516

a. Dependent Variable: MATHEMATICS.

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		.755259	.014672	51.478	.000	.727044	.784569
Intercept [subject = schid3]	Variance	.134329	.025765	5.214	.000	.092237	.195628
small3 [subject = schid3]	Variance	.096449	.026060	3.701	.000	.056794	.163791

a. Dependent Variable: MATHEMATICS.

Modeling Statistical Interaction: Two-Level BRD - 2

- Treatment at Level 1 (Random at Level 2) and Moderator at Level 2
- This Is a Cross-Level Interaction
- Suppose Treatment (T) is Dummy Variable (e.g., Small Class = 1, Else = 0) and Moderator (M) is Dummy Variable (e.g., Schools with High Proportions of Low SES Students = 1, Else = 0)
- The Simplest Way to Model the Interaction Effect Is

$$L-1 \qquad y_{ij} = \beta_{0j} + \beta_{1j}T_{ij} + \varepsilon_{ij}$$

$$L-2 \qquad \beta_{0j} = \gamma_{00} + \gamma_{01}M_j + \eta_{0j}$$

$$L-2 \qquad \beta_{1j} = \gamma_{10} + \gamma_{11}M_j + \eta_{1j}$$

Modeling Statistical Interaction: Two-Level BRD - 2

- The Mixed Effects Model is:

$$y_{ij} = \gamma_{00} + \gamma_{10}T_{ij} + \gamma_{01}M_j + \gamma_{11}T_{ij}M_j +$$
$$\eta_{0j} + T_{ij}\eta_{1j} + \varepsilon_{ij}$$

- where γ_{11} Represents the Cross-Level Two-Way Interaction and $T_{ij}\eta_{1j}$ Represents a Treatment by Level-2 (School) Interaction (Random Effect)

Example

Two-Level Model:

$$Y_{ij} = \beta_{0j} + \beta_{1j}\mathrm{SMCLASS}_{ij} + \varepsilon_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}HLOWSESCHL + \eta_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}HLOWSESCHL + \eta_{1j}$$

The Moderator Represents High Proportions of Low SES Students in Schools (> 50 Percent)

Results

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	.171825	.063389	66.664	2.711	.009	.045287	.298362
small3	.149748	.063877	58.903	2.344	.022	.021925	.277571
hlowseschl	-.490618	.094872	67.500	-5.171	.000	-.679958	-.301279
SCHLSESMALL	.094840	.095394	60.847	.994	.324	-.095922	.285601

a. Dependent Variable: MATHEMATICS.

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		.791611	.015371	51.499	.000	.762050	.822319
Intercept [subject = schid3]	Variance	.136999	.026417	5.186	.000	.093882	.199917
small3 [subject = schid3]	Variance	.102454	.027588	3.714	.000	.060440	.173673

a. Dependent Variable: MATHEMATICS.

Modeling Statistical Interaction: Three-Level BRD - 1

- Students Nested Within Classes Within Schools
- Treatment and Moderator Are at Level 1
- Treatment is Random at Levels 2 and 3 and Moderator is Fixed at Levels 2 and 3
- Suppose Treatment (T) is Dummy Variable (e.g., Within Class Grouping) and Moderator (M) is Dummy Variable (e.g., Low SES = 1, Else = 0)
- The Mixed Effects Model Is

$$y_{ijk} = \delta_{000} + \delta_{100} T_{ijk} + \delta_{200} M_{ijk} + \delta_{300} T_{ijk} M_{ijk} +$$

$$\xi_{00k} + T_{ijk} \xi_{10k} + \eta_{0jk} + T_{ijk} \eta_{1jk} + \varepsilon_{ijk}$$

- where δ_{300} Represents the Two-Way Interaction and $T_{ijk}\xi_{10k}$, $T_{ijk}\eta_{1jk}$ Represent the Treatment by Level-3 (School) and by Level-2 (Class) Interactions (Random Effects)

Modeling Statistical Interaction: Three-Level BRD - 1

$$L-1 \quad y_{ijk} = \beta_{0jk} + \beta_{1jk}T_{ijk} + \beta_{2jk}M_{ijk} + \beta_{3jk}T_{ijk}M_{ijk} + \varepsilon_{ijk}$$

$$L-2 \quad \beta_{0jk} = \gamma_{00k} + \eta_{0jk}$$

$$L-2 \quad \beta_{1jk} = \gamma_{10k} + \eta_{1jk}$$

$$L-2 \quad \beta_{2jk} = \gamma_{20k}$$

$$L-2 \quad \beta_{3jk} = \gamma_{30k}$$

$$L-3 \quad \gamma_{00k} = \delta_{000} + \xi_{00k}$$

$$L-3 \quad \gamma_{10k} = \delta_{100} + \xi_{10k}$$

$$L-3 \quad \gamma_{20k} = \delta_{200}$$

$$L-3 \quad \gamma_{30k} = \delta_{300}$$

Modeling Statistical Interaction: Three-Level BRD - 2

- Treatment at Level 2 (Random at Level 3) and Moderator at Level 1 (Fixed)
- Suppose Treatment (T) is Class Variable (e.g., Small Class = 1, Else = 0) and Moderator (M) is a Student Variable (e.g., Gender)
- The Mixed Effects Model is:

$$y_{ijk} = \delta_{000} + \delta_{010} T_{jk} + \delta_{100} M_{ijk} + \delta_{110} T_{jk} M_{ijk} +$$

$$\xi_{00k} + T_{jk} \xi_{01k} + \eta_{0jk} + \varepsilon_{ijk}$$

- where δ_{110} Represents the Two-Way Cross-Level Interaction and $T_{jk}\xi_{01k}$ Represents the Treatment by Level-3 (School) Interaction (Random Effect)

Modeling Statistical Interaction: Three-Level BRD - 2

$$L-1 \quad y_{ijk} = \beta_{0jk} + \beta_{1jk} M_{ijk} + \varepsilon_{ijk}$$

$$L-2 \quad \beta_{0jk} = \gamma_{00k} + \gamma_{01k} T_{jk} + \eta_{0jk}$$

$$L-2 \quad \beta_{1jk} = \gamma_{10k} + \gamma_{11k} T_{jk}$$

$$L-3 \quad \gamma_{00k} = \delta_{000} + \xi_{00k}$$

$$L-3 \quad \gamma_{01k} = \delta_{010} + \xi_{01k}$$

$$L-3 \quad \gamma_{10k} = \delta_{100}$$

$$L-3 \quad \gamma_{11k} = \delta_{110}$$

Modeling Statistical Interaction: Three-Level BRD - 3

- Treatment at Level 2 (Random at Level 3) and Moderator at Level 2 (Fixed)
- Suppose Treatment (T) is Dummy Variable (e.g., Small Class = 1, Else = 0) and Moderator (M) is a Teacher Variable (e.g., hours of PD)
- The Mixed Effects Model is:

$$y_{ijk} = \delta_{000} + \delta_{010} T_{jk} + \delta_{020} M_{jk} + \delta_{030} T_{jk} M_{jk} +$$
$$\xi_{00k} + T_{jk} \xi_{01k} + \eta_{0jk} + \varepsilon_{ijk}$$

- where δ_{030} Represents the Two-Way Interaction and $T_{jk}\xi_{01k}$ Represents the Treatment by Level-3 (School) Interaction (Random Effect)

Modeling Statistical Interaction: Three-Level BRD - 3

$$L-1 \quad y_{ijk} = \beta_{0jk} + \varepsilon_{ijk}$$

$$L-2 \quad \beta_{0jk} = \gamma_{00k} + \gamma_{01k} T_{jk} + \gamma_{02k} M_{jk} + \gamma_{03k} T_{jk} M_{jk} + \eta_{0jk}$$

$$L-3 \quad \gamma_{00k} = \delta_{000} + \xi_{00k}$$

$$L-3 \quad \gamma_{01k} = \delta_{010} + \xi_{01k}$$

$$L-3 \quad \gamma_{02k} = \delta_{020}$$

$$L-3 \quad \gamma_{03k} = \delta_{030}$$

Modeling Statistical Interaction: Three-Level BRD - 4

- Treatment at Level 2 (Random at Level 3) and Moderator at Level 3
- Suppose Treatment (T) is Dummy Variable (e.g., Small Class = 1, Else = 0) and Moderator (M) is a School Variable (e.g., School with High Proportions of Low SES Students = 1 Else = 0)
- The Mixed Effects Model is:

$$y_{ijk} = \delta_{000} + \delta_{010} T_{jk} + \delta_{001} M_k + \delta_{011} T_{jk} M_k + \xi_{00k} + T_{jk} \xi_{01k} + \eta_{0jk} + \varepsilon_{ijk}$$

- Where δ_{011} Represents the Two-Way Cross-Level Interaction and $T_{jk}\xi_{01k}$ Represents the Treatment by Level-3 (School) Interaction (Random Effect)

Modeling Statistical Interaction: Three-Level BRD - 4

$$L-1 \quad y_{ijk} = \beta_{0jk} + \varepsilon_{ijk}$$

$$L-2 \quad \beta_{0jk} = \gamma_{00k} + \gamma_{01k} T_{jk} + \eta_{0jk}$$

$$L-3 \quad \gamma_{00k} = \delta_{000} + \delta_{001} M_k + \xi_{00k}$$

$$L-3 \quad \gamma_{01k} = \delta_{010} + \delta_{011} M_k + \xi_{01k}$$

Power and Interaction Effects (IE)

- Tests of Interaction Effects are Typically Underpowered (Compared to Tests for Main Effects)

- Empirical Evidence in Medicine Suggests that the IE Needs to Be at Least Twice as Large as the Main Treatment Effect to Achieve Similar Power

- If Magnitude Is the Same as the Main Treatment Effect Power is Typically Smaller. Larger Sample Sizes Are Typically Needed to Detect IE

- Power in HLM (Moderators) Visit the Link

https://www.causalevaluation.org/

Power and Interaction Effects (IE): Two-Level CRD

- The Sample Size at Top Level Influences Power More than the Sample Size at the First Level
- For Level-1 Moderators that Are Fixed at Level 2 the Sample Size at Level 1 Matters as Well
- Power is Smaller for Level-2 Moderators
- Power is Higher for Lower-Level Fixed Moderators
- Level-1 and Level-2 Covariates Improve Power

Power and Interaction Effects (IE): Two-Level BRD

- The Sample Size at Top Level Influences Power More than the Sample Size at the First Level (Especially for Random Treatment Effects)

- Level-1 and Level-2 Covariates Improve Power

- Level-1 Moderators can be Treated as Random or Fixed at the Second Level. Power is Higher for Fixed Moderators

- When Level-1 Moderators Are Fixed the Sample Size at Level 1 Matters as Well

Power and Interaction Effects (IE): Three-Level CRD

- The Sample Size at Top Level Influences Power More than the Sample Sizes at Lower Levels

- When Lower-Level Moderators Are Fixed Lower-Level Sample Sizes Matter as Well

- Power is Smaller for Level-3 Moderators. Power is Larger for Level-1 Moderators

- Covariates Improve Power

- Power is Higher for Lower-Level Fixed Moderators