Designing Studies Probing Mediation

Session 21

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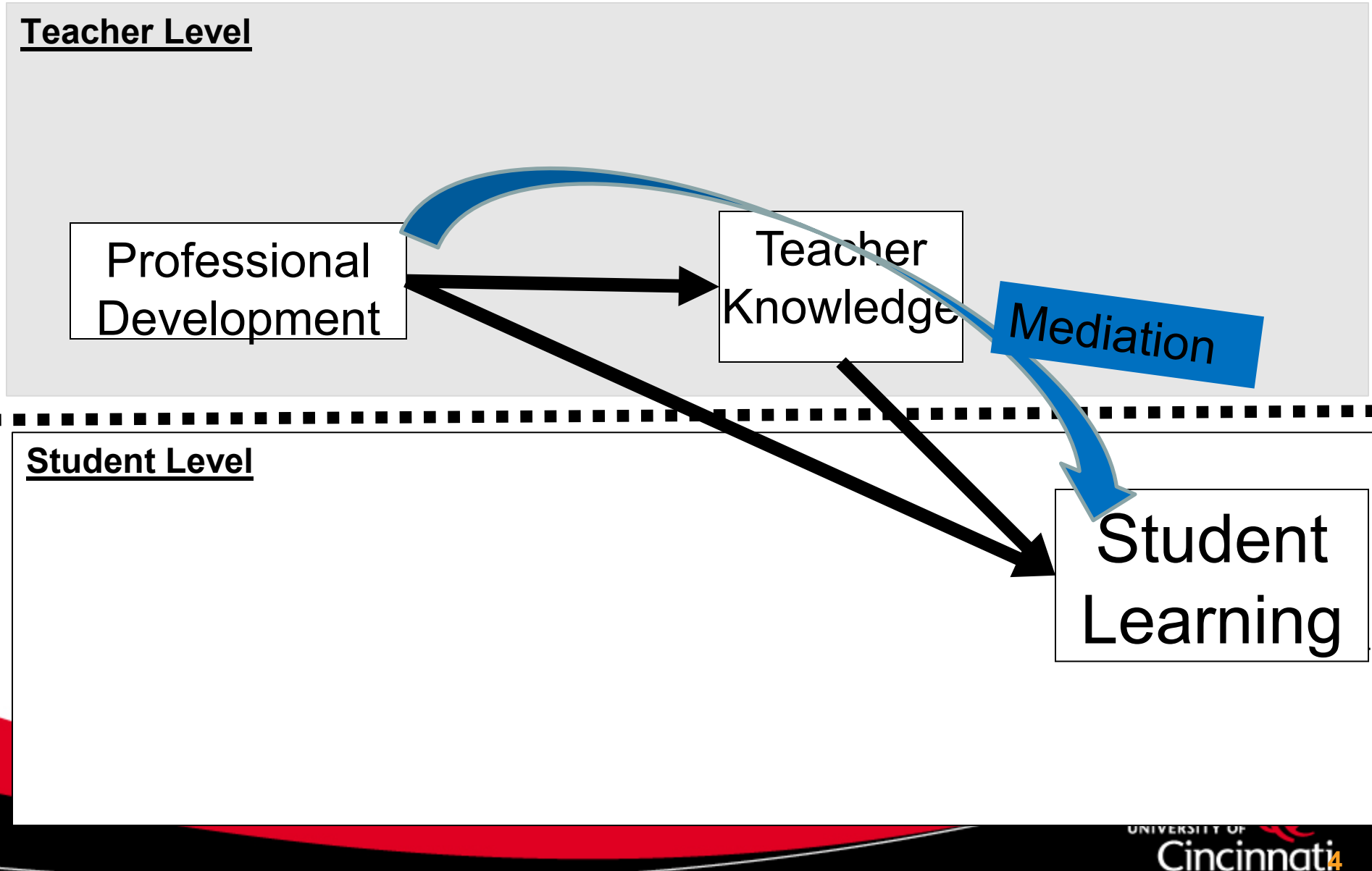
Purpose

- From Analysis to Prospective Design:
  - How might we design studies to ensure they have reasonable chance of detecting mediation effects if they exist?
    - E.g., what are reasonable sample sizes?
  - What is the requisite scale for sufficiently powered studies targeting multilevel mediation?
  - Are typical sample sizes enough?

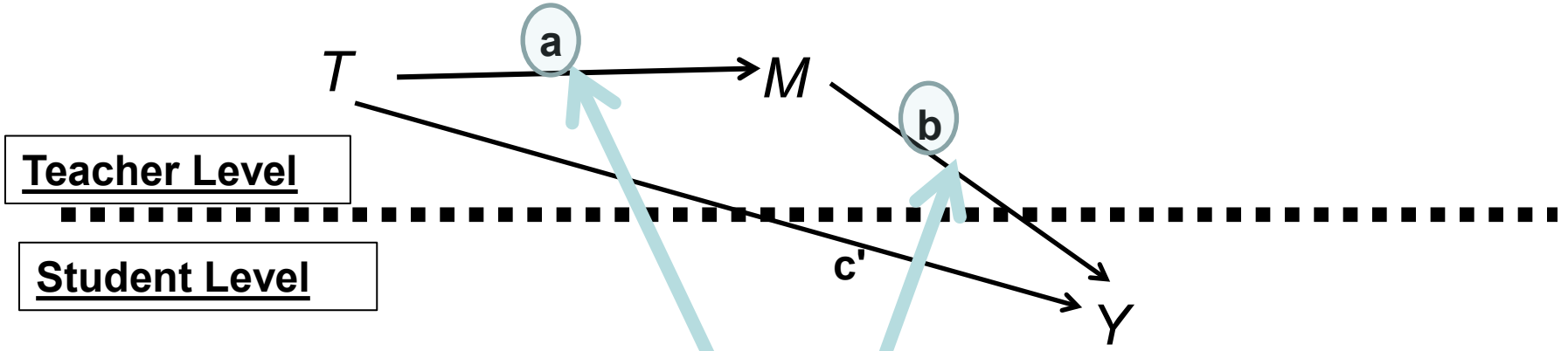
Power Analyses for Multilevel Mediation

- Simple two-level mediation example
  - Teachers are randomly assigned to participate in a PD program designed to equip teachers with core pedagogical and substantive knowledge
  - Students nested within teachers
- Outcome of interest is students' achievement
- Mediator of interest is teacher knowledge
- Goal: Design a study to detect if the impact of PD on student achievement is mediated by changes in teacher knowledge

Multilevel Mediation (2-2-1)



2-2-1 Mediation Model



Mediation model

Teacher level $M_j = \pi_0 + aT_j + \varepsilon_j^M \qquad \varepsilon_j^M \sim N(0, \sigma^2_{M|})$

Outcome model

Student level $Y_{ij} = \beta_{0j} + \varepsilon_{ij}^Y \qquad \varepsilon_{ij}^Y \sim N(0, \sigma^2_{Y|})$

Teacher level $\beta_{0j} = \gamma_{00} + bM_j + c'T_j + u_{0j} \qquad u_{0j} \sim N(0, \tau^2_{Y|})$

Parameters Governing Power for 2-2-1

J : total number of clusters

n : number of individuals per cluster

ρ : Intraclass correlation coefficient

R_{L1}^2 : proportion of variance explained at level-1

R_{L2}^2 : proportion of variance explained at level-2

P: proportion of level-2 units randomized to treatment

a: treatment-mediator path coefficient

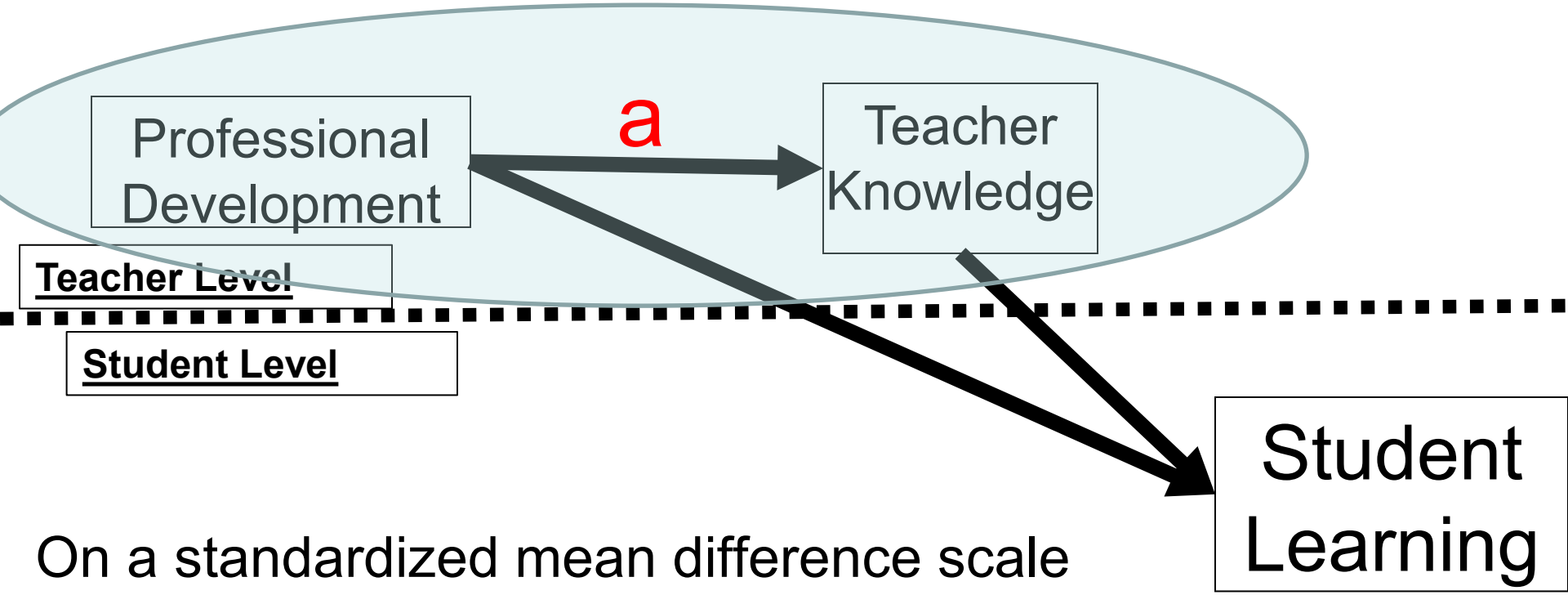
b: mediator-outcome path coefficient

c‘: direct effect of treatment on outcome

R_M^2 : proportion of mediator variance explained by covariates

Cluster-Level Mediation
a Parameter

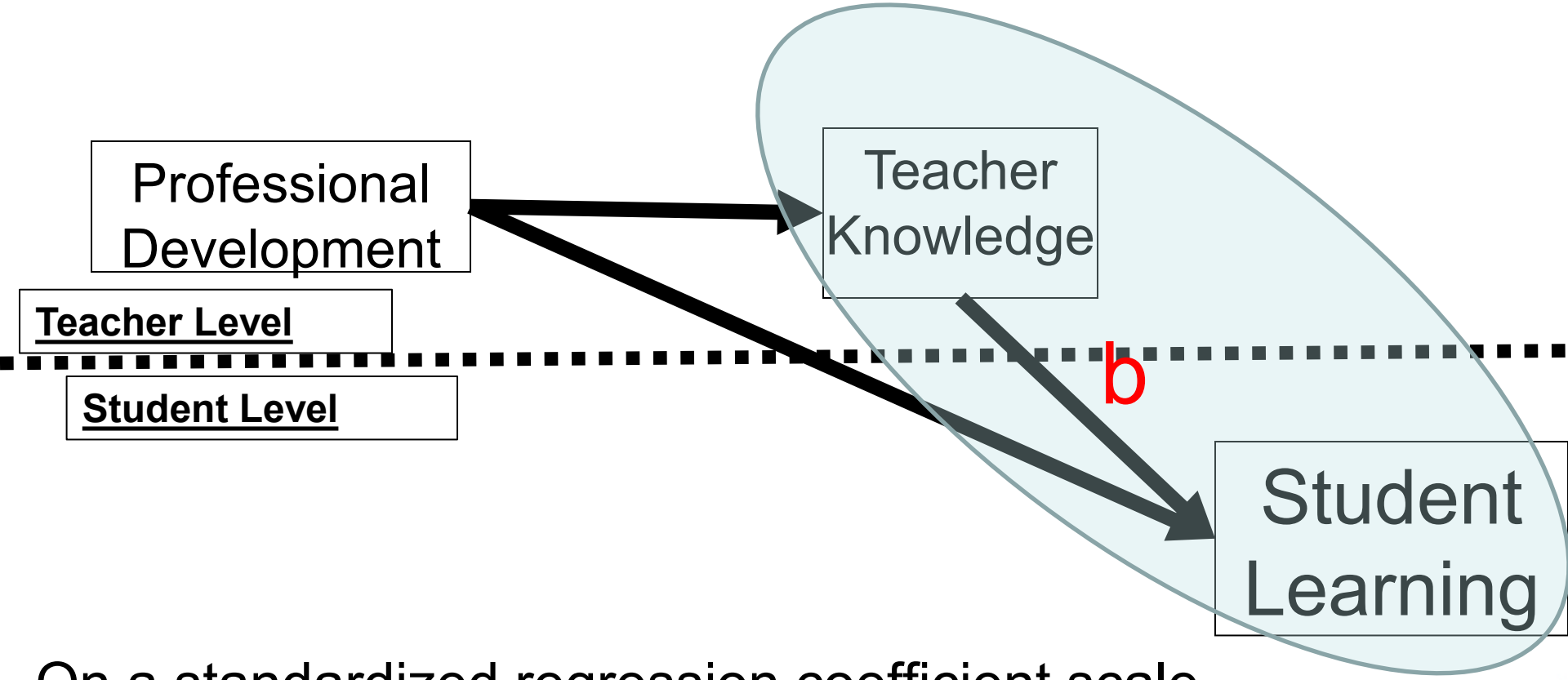
a: treatment-mediator path coefficient



On a standardized mean difference scale (same as main effect)

Cluster-Level Mediation
b Parameter

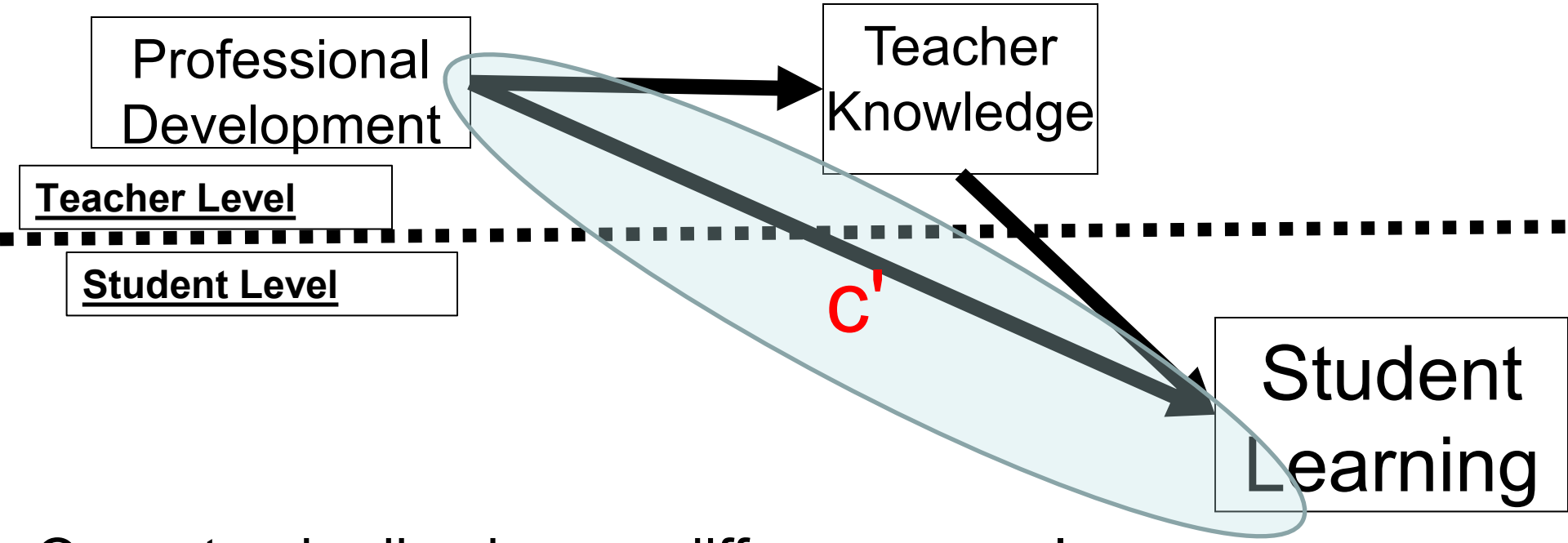
b: mediator-outcome path coefficient



On a standardized regression coefficient scale

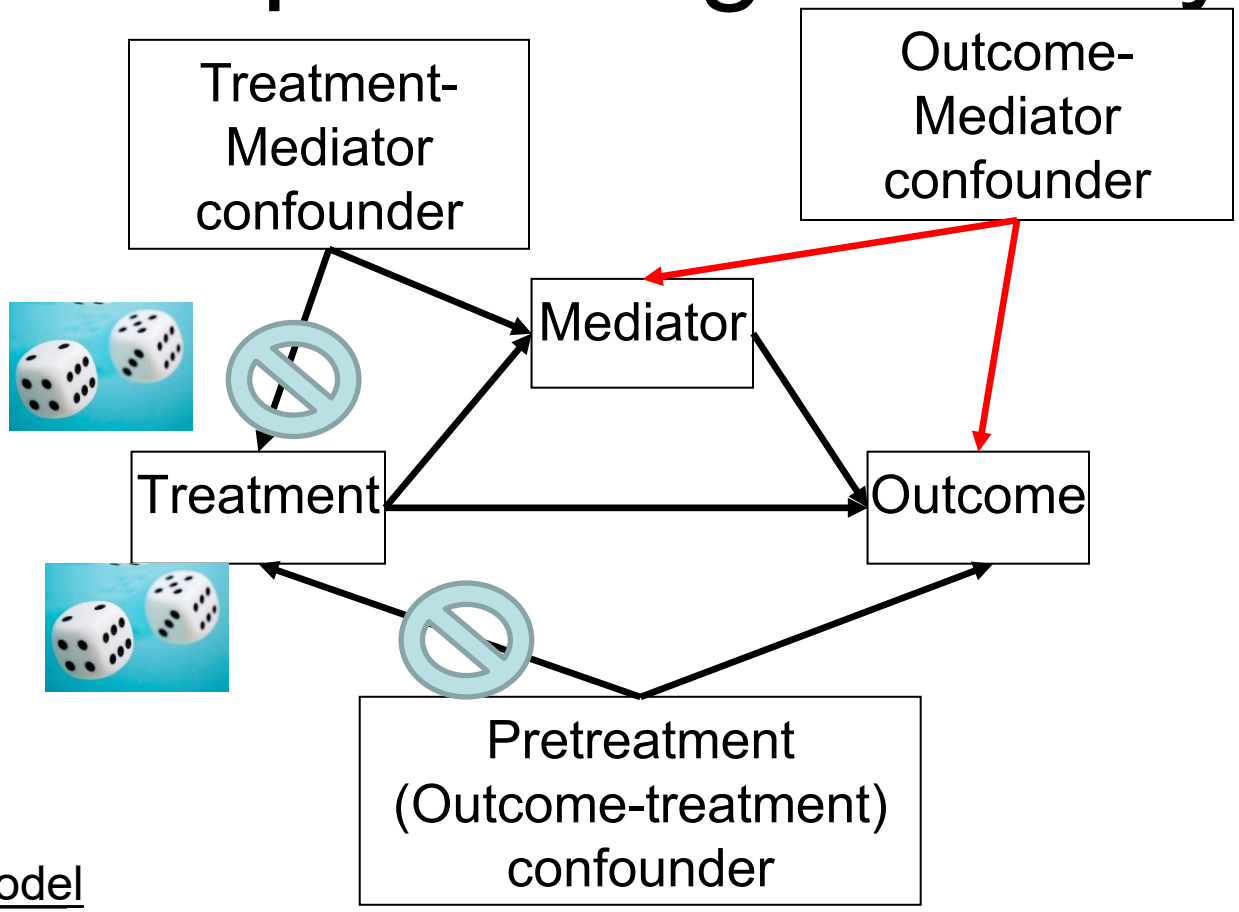
Cluster-Level Mediation
c' Parameter

c': direct effect of treatment on outcome



On a standardized mean differences scale

Sequential Ignorability



Mediation model

Class level

$$M_j = \pi_0 + aT_j + \beta_1 X_j^{L2} + \varepsilon_j^M \qquad \varepsilon_j^M \sim N(0, \sigma_{M|}^2)$$

Outcome model

Student level

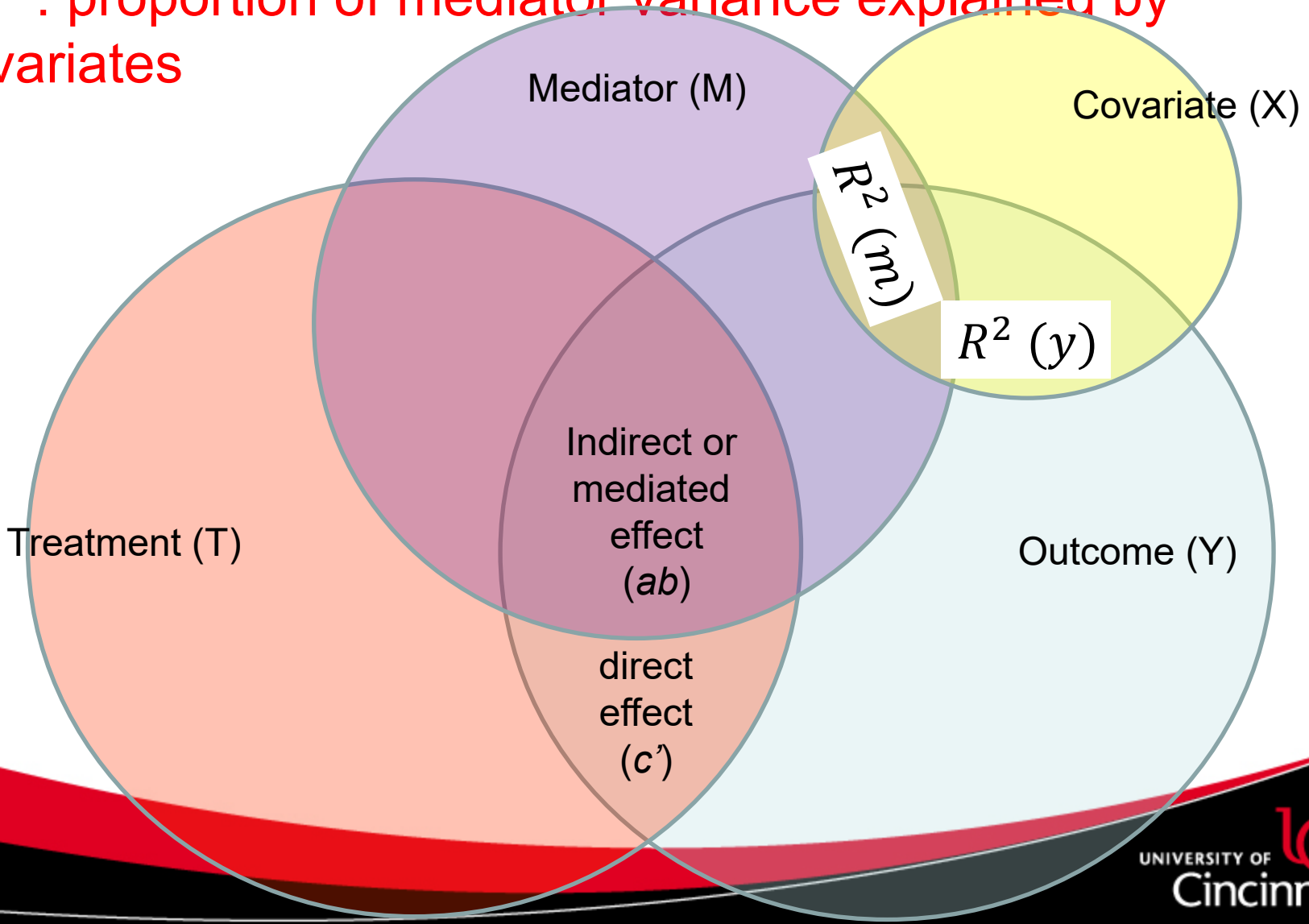
$$Y_{ij} = \beta_{0j} + \beta_{1Y} X_{ij}^{L1} + \varepsilon_{ij}^Y \qquad \varepsilon_{ij}^Y \sim N(0, \sigma_{Y|}^2)$$

Class level

$$\beta_{0j} = \gamma_{00} + bM_j + c'T_j + \gamma_1 X_j^{L2} + u_{0j} \qquad u_{0j} \sim N(0, \tau_{Y|}^2)$$

Cluster-Level Mediation
R_M^2

R_M^2 : proportion of mediator variance explained by covariates



Scale of Effect Size

- Lots of different approaches
  - Review lit and identify most meaningful for your context

- One simple approach: Multiply *a* and *b* paths where the magnitude of the paths is based on common (theoretical or empirical) effect size interpretations

Effect Size

- *a* path
  - standardized mean difference scale for dichotomous treatments
- *b* path
  - If the mediator and outcome are standardized, its on a standardized regression coefficient scale (controlling for treatment and covariates)
- Then effect size is just product of a and b

Possible (theoretical) Benchmarks

- Dichotomous treatment, continuous mediator

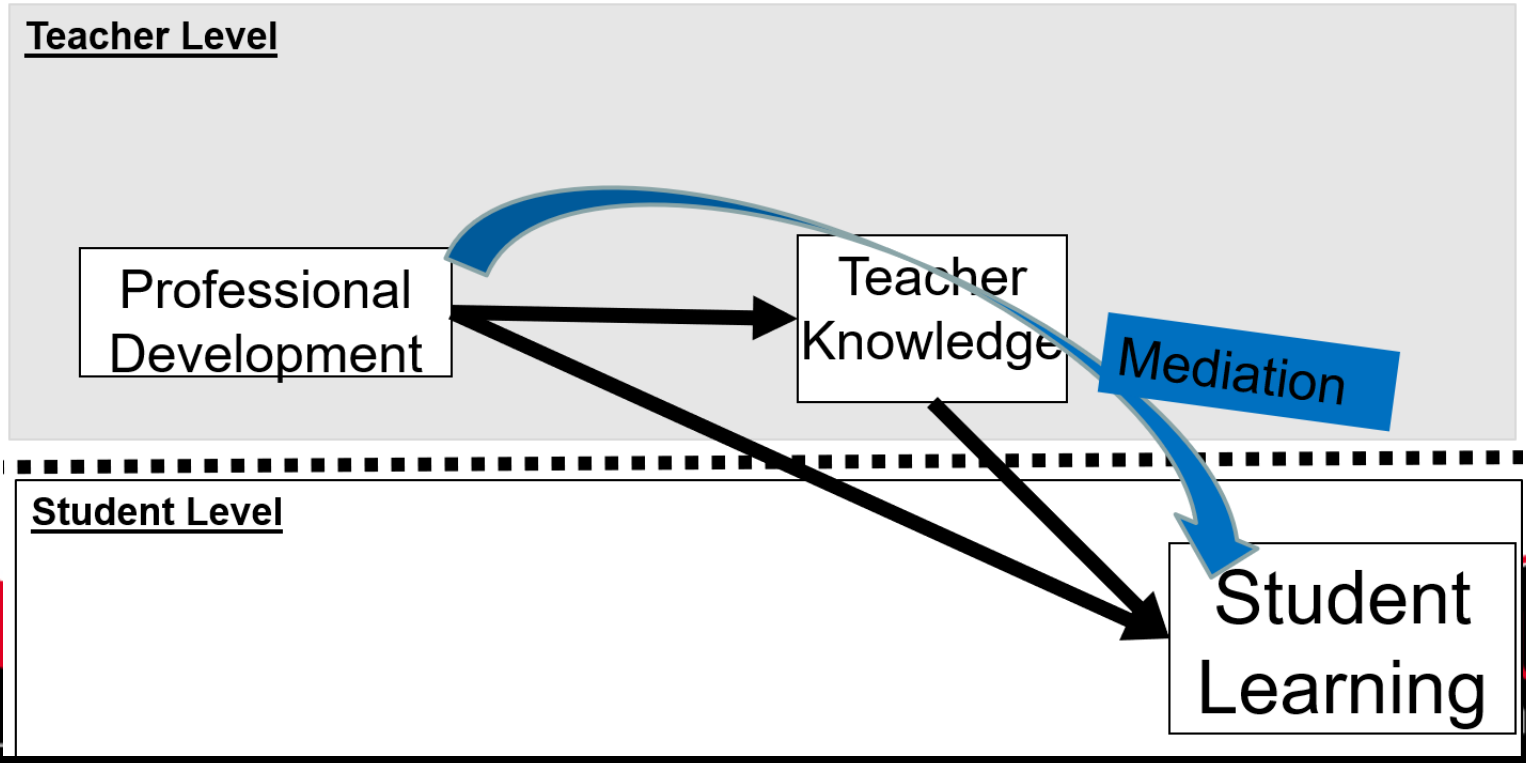
Size	=	XX (i.e., effect of a * effect of b)
– Small	=	.02 (i.e., .2*.1)
– Medium	=	.15 (i.e., .5*.3)
– Large	=	.40 (i.e., .8*.5)

Tests of Mediation

- Some Common Tests of mediation
  - Sobel test
  - Test of joint significance
  - Monte Carlo interval test
  - Bootstrap resampling

Example Power Analysis

Consider a professional development program that aims to improve student learning by improving teacher knowledge. Assume teachers are randomly assigned to participate in the professional development program or a control condition. If we plan to sample about 20 teachers per school, how many schools do we need for an 80% chance of detecting a mediation effect? (more info on next slide)



Example: Cluster-Level Mediation Parameters

a: treatment-mediator path coefficient — 0.5

b: mediator-outcome path coefficient — 0.3

c‘: direct effect of treatment on outcome — 0.1

ρ : Intraclass correlation — 0.15

R_{L1}^2 : proportion of variance explained at level-1 — 0.5

R_{L2}^2 : proportion of variance explained at level-2 — 0.5

R_M^2 : proportion of mediator variance explained by covariates — 0.5

P: proportion of level-2 units randomized to treatment — 0.50

J : total number of clusters — 40

n : number of individuals per cluster — 20

PowerUpR Shiny App

https://powerupr.shinyapps.io/index/

Results

```
Statistical power:
-------------------------------------
       t sobel joint  mc
a  0.793    NA    NA  NA
b  1.000    NA    NA  NA
ab    NA 0.751 0.793 0.8
-------------------------------------
Degrees of freedom for path a: 52
Degrees of freedom for path b: 52
Standardized standard error for path a: 0.177
Standardized standard error for path b: 0.041
Type I error rate: 0.05
Two-tailed test: TRUE
```

Exercise

Program: A new math curriculum for 3rd graders. The curriculum is implemented at the school level and expected to impact student outcomes by improving school atmosphere. The researchers plan to randomly assign schools to the treatment or control condition and assess school atmosphere at the school level during the study.

RQ1: Is the new math curriculum more effective than the traditional one (main effect)?

RQ2: Does the curriculum operate through changes in the atmosphere as theorized (mediation effect)?

Design: How large of a sample do we need to detect a mediation effect with 80% power?

Example parameter values on next slide...

2-2-1 Mediation Parameters

J : total number of clusters—??

n : number of individuals per cluster—50

ρ : Intraclass correlation—0.25

a: treatment-mediator path coefficient—0.5

b: mediator-outcome path coefficient—0.25

c‘: direct effect of treatment on outcome—0.1

R_M^2 : proportion of mediator variance explained by covariates—0.3

R_{L1}^2 : proportion of variance explained at level-1—0.4

R_{L2}^2 : proportion of variance explained at level-2—0.5

P: proportion of level-2 units randomized to treatment—0.5

PowerUpR Shiny App

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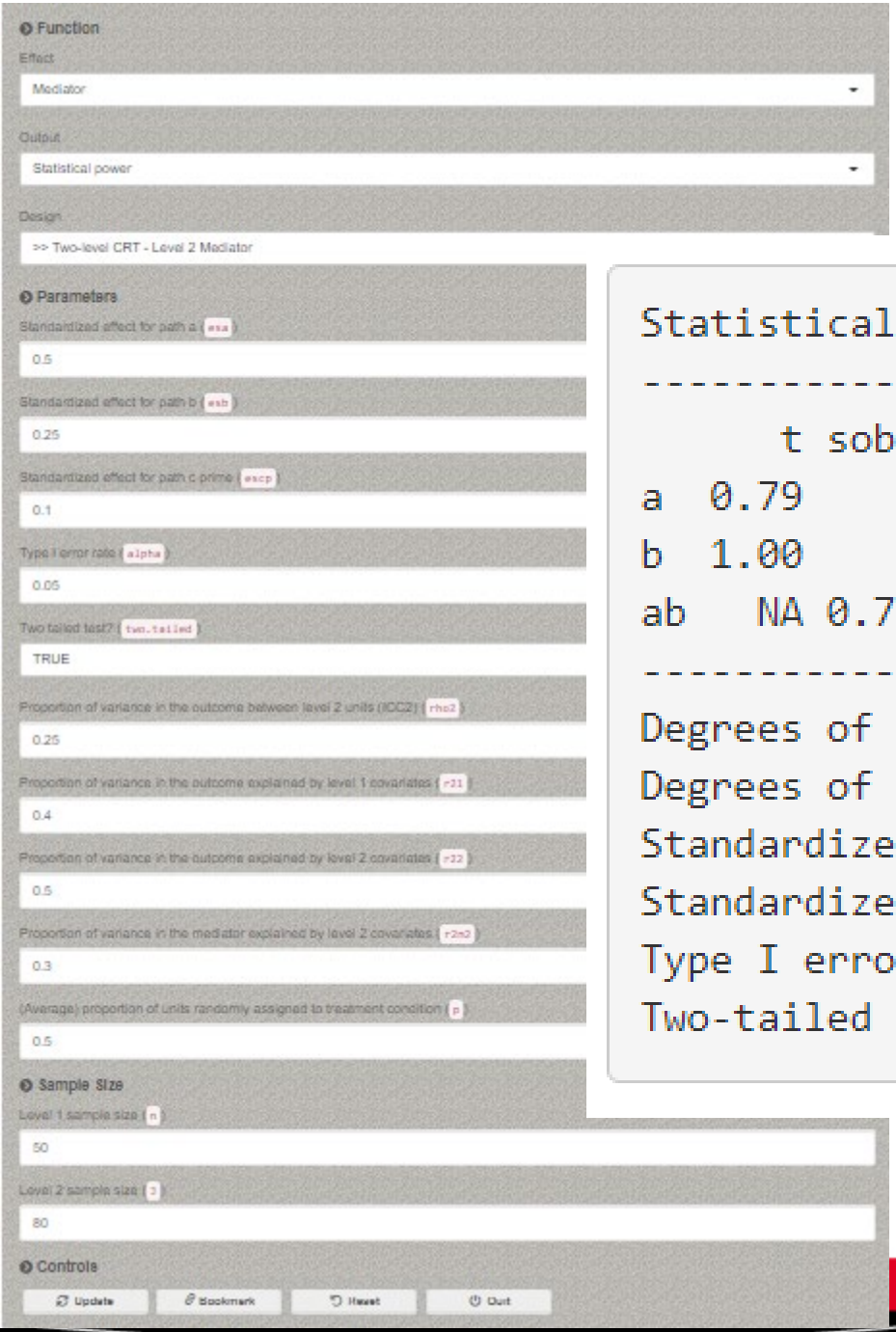
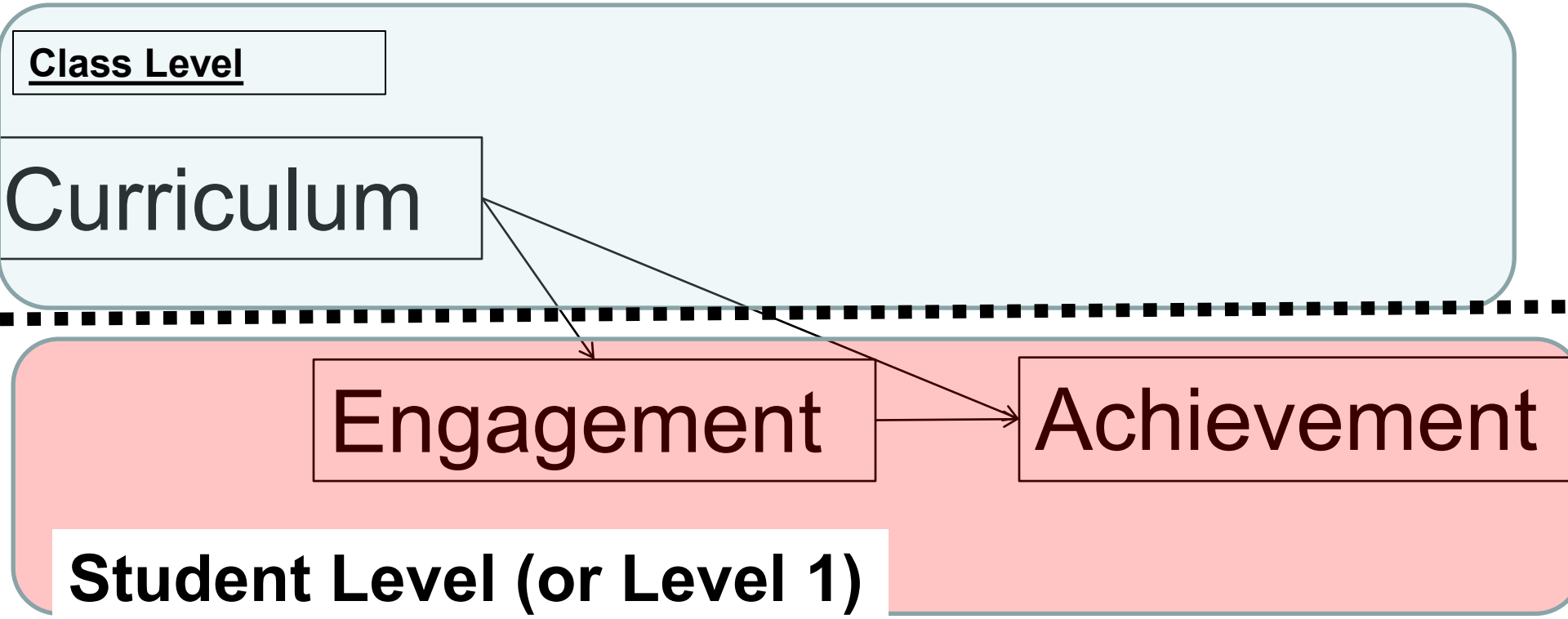



```
Statistical power:
-------------------------------------
      t sobel joint    mc
a  0.79    NA    NA    NA
b  1.00    NA    NA    NA
ab   NA 0.724  0.79 0.809
-------------------------------------
Degrees of freedom for path a: 76
Degrees of freedom for path b: 76
Standardized standard error for path a: 0.179
Standardized standard error for path b: 0.04
Type I error rate: 0.05
Two-tailed test: TRUE
```

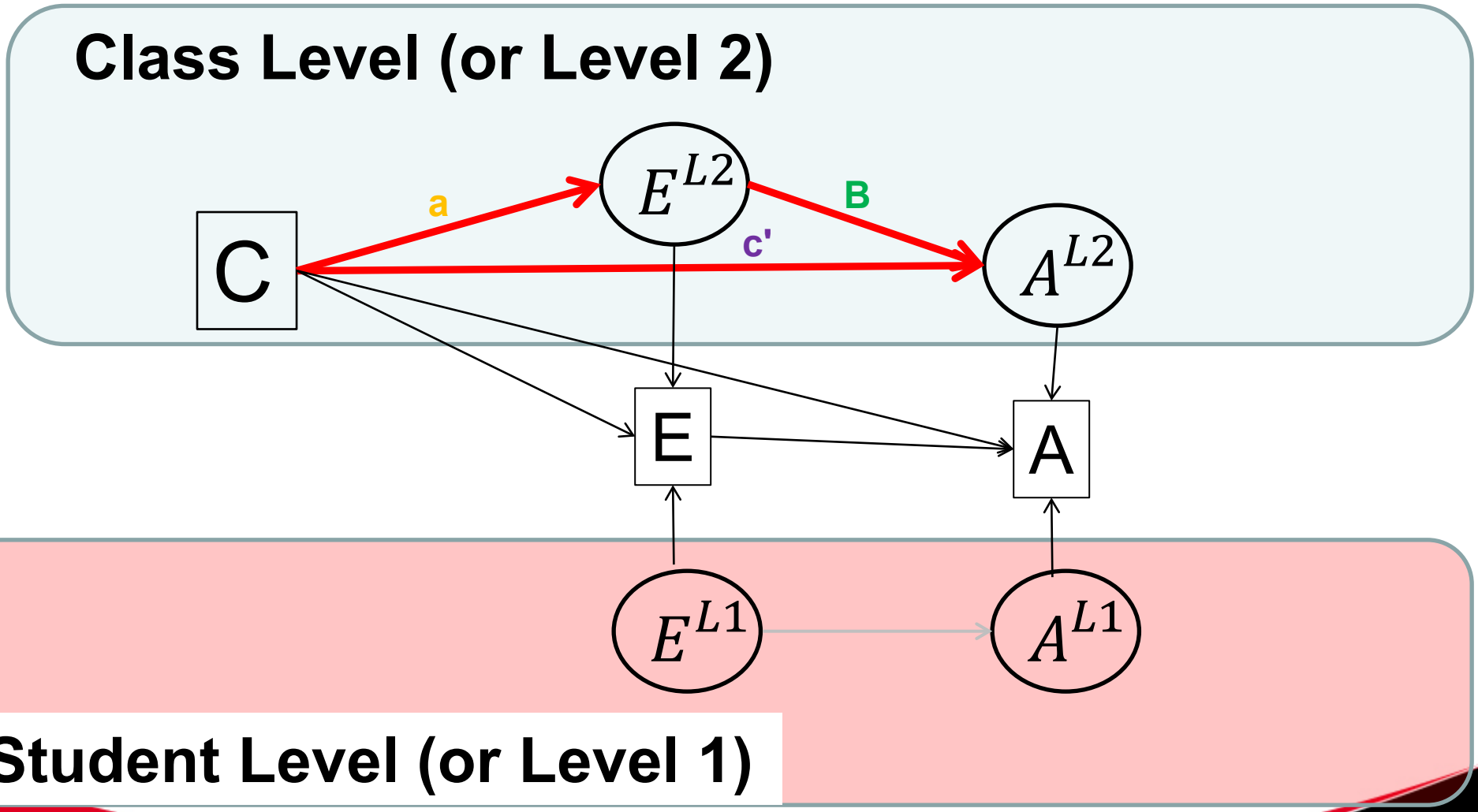
2-1-1 Power Analysis Example

- Consider a simple two-level mediation example with students nested within classes that are randomly assigned to participate in an innovative curriculum designed to engage students of all levels
- Let the outcome of interest be students' achievement and assume that the mediator of interest is student engagement
- We are interested in designing a study to detect the extent to which the impact of participating in the innovative curriculum on student achievement is mediated by changes in (individual and collective) student engagement

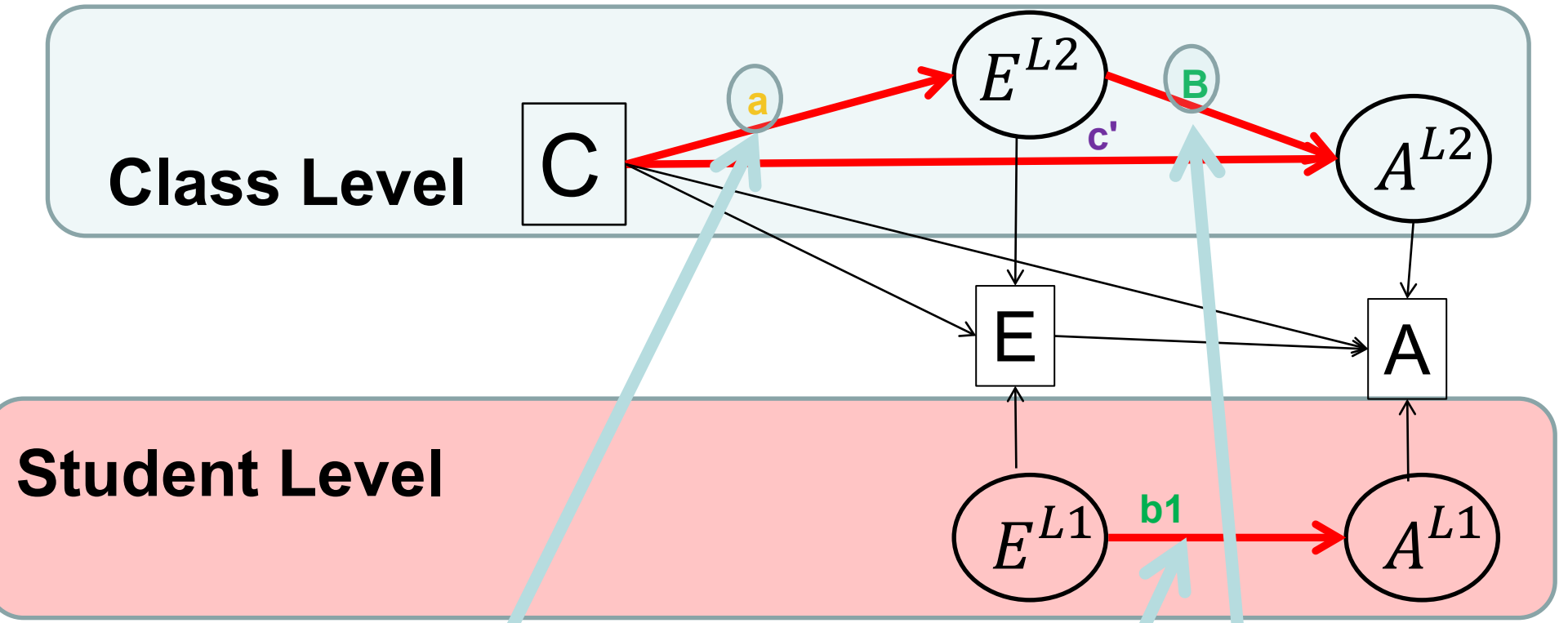
Graphical Illustration of 2-1-1 Mediation



Classroom Engagement



Classroom Engagement



Mediator Model

$$E_{ij} = \beta_{0j} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + aC_j + u_{0j}$$

Outcome Model

$$Y_{ij} = \beta_{0j} + b_1 E^{L1} + r_{ij}$$
$$\beta_{0j} = \gamma_{00} + c' C_j + B E^{L2} + u_{0j}$$

Parameters for designing 2-1-1 mediation studies

Alpha: type 1 error rate (2 tailed): 0.05

a: treatment-mediator relationship effect size: 0.5

b1: mediator-outcome relationship at L1 effect size: 0.4

B: total mediator-outcome relationship effect size (B=b1+b2): 0.4

c': direct effect of treatment on outcome effect size: 0.1

rho2: Intraclass correlation for outcome: 0.2

rhom2: Intraclass correlation for mediator: 0.2

R21: outcome variance explained by covariates at L1: 0.5

R22: outcome variance explained by covariates at L2: 0.5

R2m1: mediator variance explained by covariates at L1: 0.5

R2m2: mediator variance explained by covariates at L2: 0.5

P: proportion of clusters in treatment: 0.5

n: L1 sample size: 20

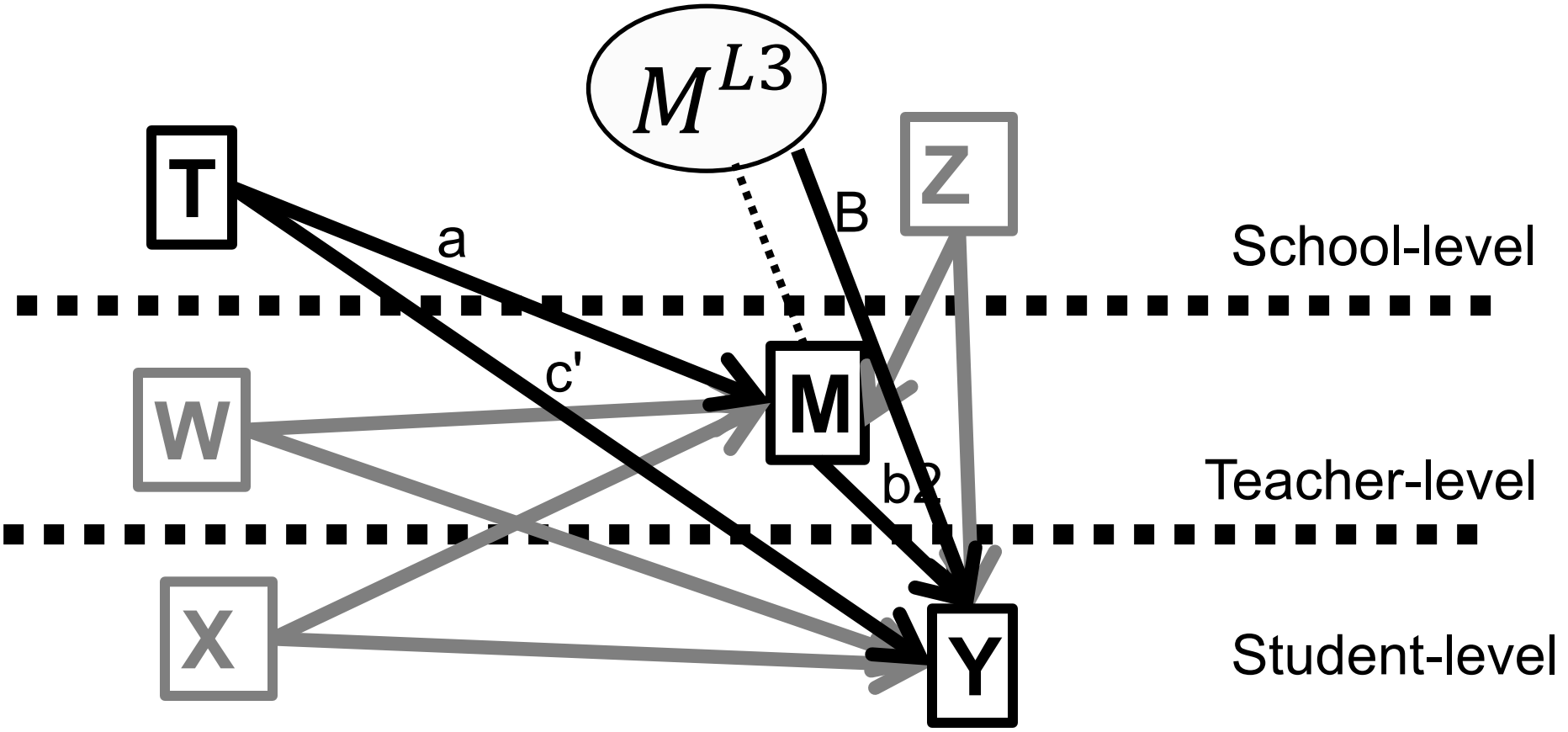
J: L2 sample size: ??

Power: 80%

3-2-1 Example

- School-randomized design
  - students nested within classrooms nested within schools
- Treatment: teacher professional development (assigned at school level)
- Outcome: students' achievement
- Mediator: teacher instruction
- Goal: 3-2-1 mediation
  - We are interested in designing a study to detect the extent to which the impact of participating in the PD program on student achievement is mediated by changes in instruction

Three-Level Example: 3-2-1



3-2-1 Parameters

$a = 0.50$ (treatment-mediator relationship [Cohen's d scale])

$B = 0.30$ (mediator-outcome relationship [Standardized regression scale])

$v_Y^2 = 0.10$ (unconditional outcome variance at school-level)

$\tau_Y^2 = 0.10$ (unconditional outcome variance at class-level)

$\sigma_Y^2 = 0.80$ (unconditional outcome variance at individual-level)

$\tau_M^2 = 0.20$ (unconditional outcome variance at school-level)

$\sigma_M^2 = 0.80$ (unconditional outcome variance at class-level)

$R_{Y^{L3}}^2 = R_{Y^{L2}}^2 = R_{Y^{L1}}^2 = 0.50$ (outcome variance explained at each level)

$R_{M^{L3}}^2 = R_{M^{L2}}^2 = 0.50$ (mediator variance explained at each level)

$P = 0.50$ (proportion of schools receiving treatment)

$n_2 = 4$ (classrooms/school)

$n_1 = 20$ (students/classroom)

End of Session 21

- Break until 130pm

- Questions, Comments, & Feedback
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